Section 3.1: Relations and Functions

- A **relation** is any set of ordered pairs in the form: (x, y).
- The **domain** of a relation is all the $x - values$ in the ordered pairs that make up the relation.
- The **range** of a relation is all the $y - values$ in the ordered pairs that make up the relation.
- A **function** is a relation that assigns to each element in its domain *exactly one* element in the range.
  A relation is not a function if any points have the same x-value with different y-values
  A relation is a function if no points have the same x-value with different y-values
- If the domain of a **function** consists of $x - coordinate$s and the range consists of $y - coordinates$ we say y **is a function of** x.

For Example:

- Find the domain and range of the relation listed below.
- Determine whether the relation is a function or not a function.

Relation R = { (5,1) (5,3) (4,7) (6, 8) (2,8) }

- The domain is the $x - coordinates$ of the points.
- The domain of a relation that consists of a finite number of points is usually represented using set braces.
- The range is the $y - coordinates$ of the points.
- The range of a relation that consists of a finite number of points is usually represented using set braces.
- Each element in the domain and range only needs to be listed once
- You can write the elements of the domain or range in any order. I usually write the elements in ascending order.

Answer:

Domain {2,4,5,6}

Range {1,3,7,8}

This relation is not a function because there are two points { (5, 1) and (5,3)}that have the same x and have different y's.

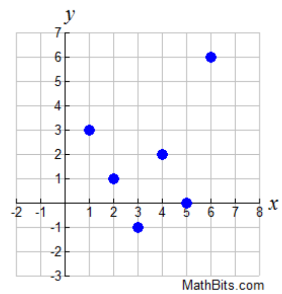
For Example:

- The relation graphed below is named R, Create the points implied by the relation.

- Find the domain and range of the relation listed below.

- Determine whether the relation is a function or not a function.

In this example the ordered pairs are graphed as opposed to listed individually.



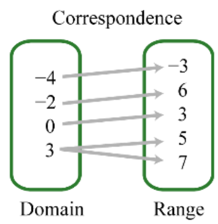
- The domain is the x-coordinates of the points.
- The domain of a relation that consists of a finite number of points are usually represented using set braces.
- The range is the y-coordinates of the points.
- The range of a relation that consists of a finite number of points is usually represented using set braces.

Answer:
Points of relation R:
R = { (1,3) (2,1) (3,-1) (4,2) (5,0) (6,6)}

Domain {1, 2, 3,4, 5, 6} Range {-1, 0, 1, 2, 3, 6}

This relation is a function as no two points that have the same x-value with different y-values. (We can say y is a function of x.)

For Example:



- Call the above relation R, and create the points implied by the relation
- Find the domain and range of the relation listed below.
- Determine whether the relation is a function or not a function

Answer:
Create points by following the arrows. Make the numbers in the first column “x” and the second column “y”.

R = { (-4, -3) (-2, 6) (0, 3) (3, 5) (3, 7)}

Domain = {-4, -2, 0, 3}
Range = {-3, 3, 5, 6, 7}

This relation is not a function (since the points (3,5) and (3,7) have the same x with different y’s.)

For Example:

x	y
0	0
1	1
2	4
3	9
4	16

- Call the above relation R, and create the points implied by the relation
- Find the domain and range of the relation listed below.
- Determine whether the relation is a function or not a function

Answer:

R = { (0,0) (1, 1) (2, 4) (3, 9) (4, 16)}

Domain = {0, 1, 2, 3, 4}
Range = {0, 1, 4, 9, 16}

The relation is a function (since no points have the same $x - value$ with a different $y - value$)

For Example:
Which of the following relations represent y as a function of x?

x	y
2	3
1	4
1	5
0	6

Answer:

y is NOT a function of x

(1,4) and (1,5) are points of the relation with the same x, but different y.

x	y
2	3
1	3
0	3
-1	3

Answer: y is a function of x

A function can have duplicate y's, but it cannot have duplicate x's.

R = { (-4,2)   (5,-1) (6,-3) }

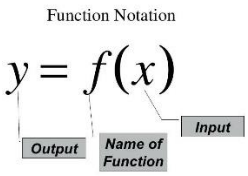
Answer: y is a function of x

R = {(1,2) (3,4) (5,6) (7,8) (3,9)}

Answer: y is not a function of x as there are duplicate x's.

The points (3,4) and (3,9) make this relation not a function.

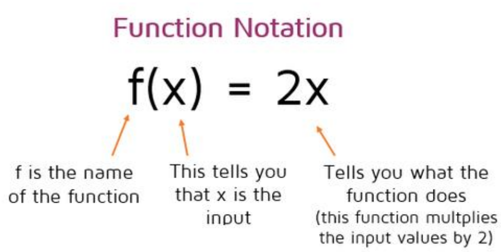
Function Notation: For the function $y = f(x)$



- We read $f(x)$ as:
  f of *x*
  or: the value of *f* at *x*.
- NOTE $f(x)$ does not mean multiplication: that is $f(x)\ does\ not\ mean\ f \times (x)$
- *f* is the name of the function
- *x* is the domain (input) value
- y is the range (output) value
  The range of a function consists of all the possible y's ($f(x)'s$) that can be generated from the x's in the domain of the function.
- The variable y and the function symbol $f(x)$ are usually interchangeable in most problems. When I see function notation I think "y".

For Example:

- The function described below is named "f"
- The variable in the parenthesis is the "domain / input" variable.
  x is the domain variable
- 2x is the range (output) value
- The $f(x)$ symbol works like a y
  In fact, the graphs of $f(x) = 3x + 1\ and\ y = 3x + 1$ are identical



Independent and Dependent Variables: For the function $y = f(x)$

- *x* is the independent variable as it can be any value in the domain
- *y* is the dependent variable as its value depends on *x*

Consider the function $y = f(x)$

x	**f(x)**
–2	–8
–1	0
0	0
1	–2
2	0
3	12

How to answer questions about a function when given a function defined in table form:

- Evaluate the function at $x = given\ number$
  Asks to find the $y - coordinate$ that relates to the of the point that has the given number as its $x - coordinate$.

- $f(given\ number)$
  Asks to find the $y - coordinate$ that relates to the of the point that has the given number as its $x - coordinate$. This is a**sking the same question as above.** This is the more common way to ask this question.

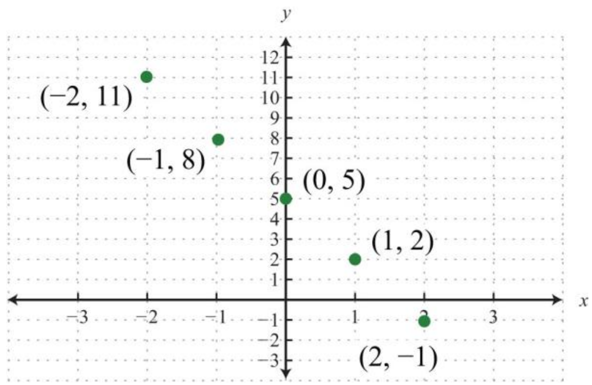
- Find all values of x such that $f(x) = given\ number$
  Wants the $x - coordinate$ of all points that have the given number as a $y - coordinate$.

Find the following:

- The domain of f
- The range of the f
- Evaluate the function at $x = -1$
- $f(-1)$
- Evaluate the function at $x = 3$
- $f(3)$
- all values of x such that $f(x) = -8$
- all values of x such that $f(x) = 0$

Answer:

- Domain = {-2, -1, 0, 1, 2, 3} (all the x-values)
- Range = {-8, -2, 0, 12} (I only need to write each value once) (all the y-values)
- Evaluate the function at $x = -1$
  Answer: $0\ or\ f(-1) = 0$.
- $f(-1) = 0$
- Evaluate the function at $x = 3$
  Answer: $12\ or\ f(3) = 12$
- $f(3) = 12$
- All values of x such that $f(x) = -8$, this is asking me to find the $x - coordinate$ for any value of x that corresponds to -8 in the $f(x)\ column$ Answer: $x = -2$
- All values of x such that $f(x) = 0$
  Answer: $x = -1, 0\ and\ 2$



How to answer questions about a function when given a function defined in table form:

- $f(given\ number)$
  Asks to find the $y - coordinate$ that relates to the point that has the given number as its $x - coordinate$.

- Find all values of x such that $f(x) = given\ number$
  Wants the $x - coordinate$ of all points that have the given number as a $y - coordinate$.

Consider the function $y = f(x)$ graphed in the left panel.

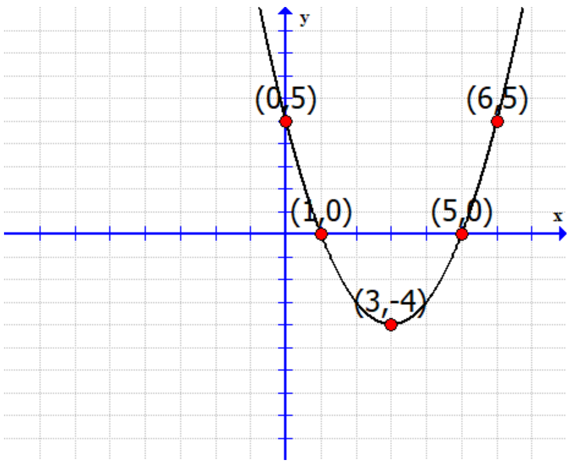
Find the following:

- The domain of f
- The range of the f
- $f(-1)$
- $f(-2)$
- all values of x such that $f(x) = 2$
- all values of x such that $f(x) = 5$

Answer:

- Domain = $\{-2, -1, 0, 1, 2\}$
  (all the $x - values$)
- Range = $\{-1, 2, 5, 8, 11\}$
  (all the $y - values$)
- $f(-1) = 8$
  (the $y - coordinate$ of the point that has $x = -1$
- $f(-2) = 11$
  (the $y - coordinate$ of the point that $has\ x = -2$)
- all values of x such that $f(x) = 2$:
  $Answer: x = 1$
  (the $x - coordinate$ of any point that has a $y = 2$)
- all values of x such that $f(x) = 5$:
  $x = 0$
  (all the $x - coordinates$ of any point that has $y = 5$)

For Example: This is a graph the function named: g(x)



How to answer questions about a function when given a function defined in table form:

- $g(given\ number)$
  Asks to find the $y-coordinate$ that relates to the point that has the given number as its $x-coordinate$.

- Find all values of x such that
  $g(x)\ =\ given\ number$
  Wants the $x-coordinate$ of all points that have the given number as a $y-coordinate$.

Find the following:

- $g(6)$
- $g(0)$
- all values of x such that $g(x)\ =\ -4$
- all values of x such that $g(x)\ =\ 5$

Answer: $g(6) = 5$
(the y-coordinate of all points that have $x\ =\ 6$)

$g(0) = 5$
(the y-coordinate of all points that have $x\ =\ 0$)

all values of x such that $g(x)\ =\ -4$: $x\ =\ 3$
(the $x-coordinate$ of all points that have a $y-coordinate\ of\ \ y\ = -4$)

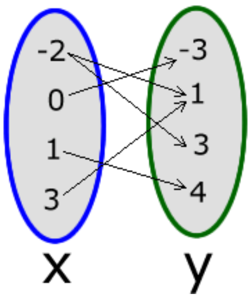
all values of x such that $g(x)\ =\ 5$: $x\ =\ 0, 6$
(the $x-coordinate$ of all points that have a $y-coordinate\ of\ y\ =\ 5$)

For Example: Let $f(x) = 3x^2 + 5x - 4$ Find the following: • $f(2)$ • $f(-1)$ • $f(a)$ Each of these asks me to evaluate the function at the value given inside the parenthesis. ***<u>This does not ask you to multiply by the value inside the parenthesis.</u>***	$f(2)$ is just asking me to evaluate the function at at $x = 2$ $f(2) = 3(2)^2 + 5(2) - 4$ $f(2) = 3(4) + 5(2) - 4$ $f(2) = 12 + 10 - 4$ Answer: $f(2) = 18$ $f(-1) = 3(-1)^2 + 5(-1) - 4$ $f(-1) = 3(1) + 5(-1) - 4$ $f(-1) = 3 + (-5) - 4$ Answer: $f(-1) = -6$ $f(a) = 3(a)^2 + 5(a) - 4$ (note $(a)^2 = a^2$ and $5(a) = 5a$) Answer: $f(a) = 3a^2 + 5a - 4$ There is nothing that should be done with this answer. It would be wrong to set this answer equal to zero and solve for x.

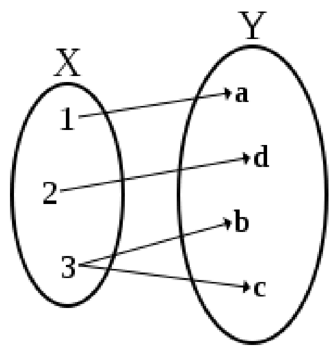
Section 3.1: Relations and functions

#1-4: Find the following:
a) Create the points implied by the relation.
b) Find the domain and range of the relation listed below.
c) Determine whether y is a function of x.

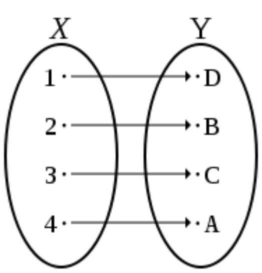
1)



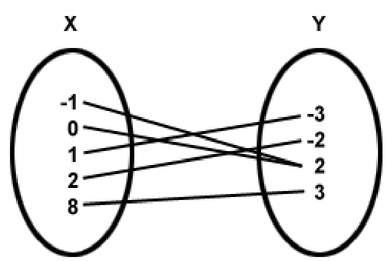
2)



3)



4)

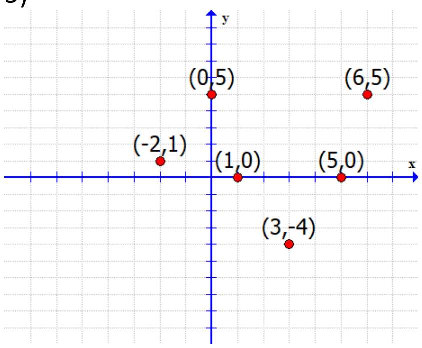


#5-8: Find the following:

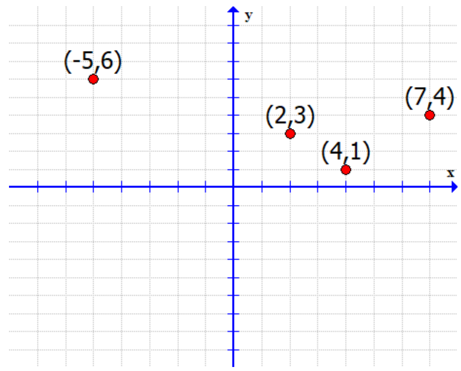
a) Find the domain and range of the relation listed below.

b) Determine whether the if y is a function of x.

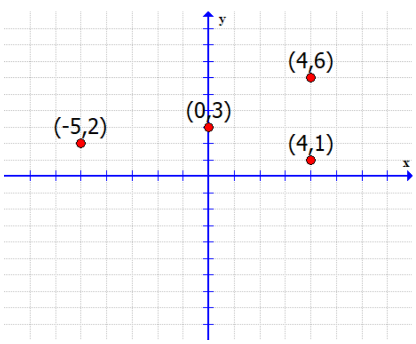
5)



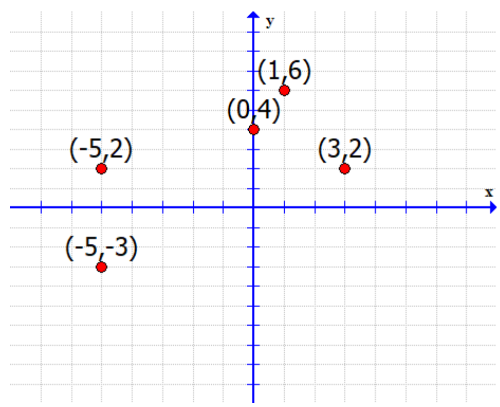
6)



7)



8)



9) Find the following: $f = \{(3,-2),\ (5,6),\ (7,3),\ (1,-2), (4,1),\ (6,7)\}$

a) The domain of f
b) The range of the f
c) $f(3)$
d) $f(1)$
e) all values of x such that $f(x) = -2$
f) all values of x such that $f(x) = 6$

10) Find the following: $f = \{(1,-3),\ (2,-3),\ (-4,2),\ (5,-2), (-3,5),\ (6,7)\}$

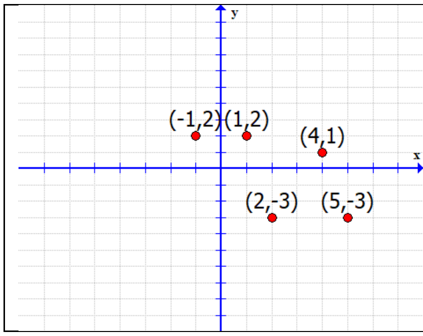
a) The domain of f
b) The range of the f
c) $f(2)$
d) $f(5)$
e) all values of x such that $f(x) = -3$
f) all values of x such that $f(x) = 2$

11) Find the following: $g = \{(9,2)\ (1,9)\ (4,1)\ (2,4)\ (6,1)\}$

a) The domain of g
b) The range of the g
c) $g(9)$
d) $g(4)$
e) all values of x such that $g(x) = 9$
f) all values of x such that $g(x) = 1$

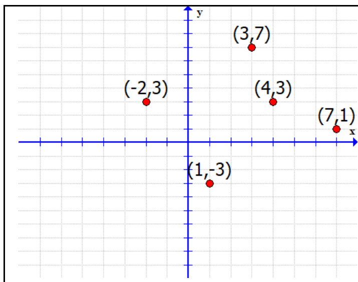
12) Find the following: $g = \{(5,-3)\ (-3,2)\ (2,-3)\ (1,2)\ (6,1)\}$

a) The domain of g
b) The range of the g
c) $g(1)$
d) $g(-3)$
e) all values of x such that $g(x) = -3$
f) all values of x such that $g(x) = 1$



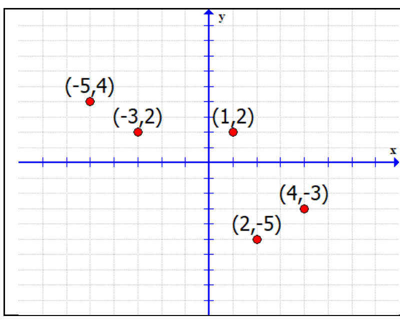
13) Given the graph of f(x), find the following:

a) The domain of f
b) The range of the f
c) $f(2)$
d) $f(1)$
e) all values of x such that $f(x) = 2$
f) all values of x such that $f(x) = -3$



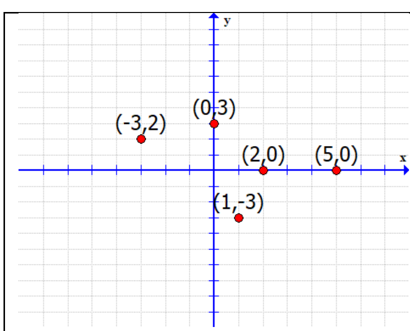
14) Given the graph of f(x), find the following:

a) The domain of f
b) The range of the f
c) $f(1)$
d) $f(3)$
e) all values of x such that $f(x) = 3$
f) all values of x such that $f(x) = 1$



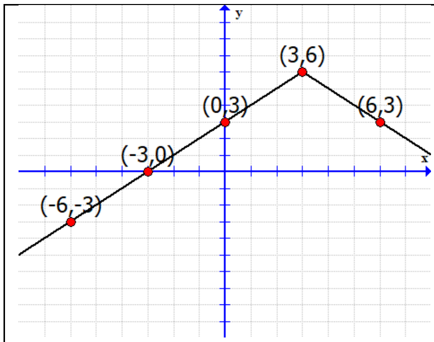
15) Given the graph of g(x), find the following:

a) The domain of g
b) The range of the g
c) $g(2)$
d) $g(4)$
e) all values of x such that $g(x) = 4$
f) all values of x such that $g(x) = -5$



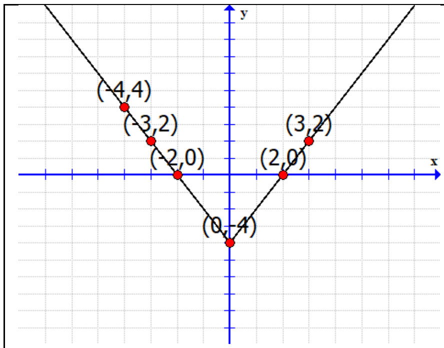
16) Given the graph of g(x), find the following:

a) The domain of g
b) The range of the g
c) $g(-3)$
d) $g(0)$
e) all values of x such that $g(x) = 0$
f) all values of x such that $g(x) = 2$



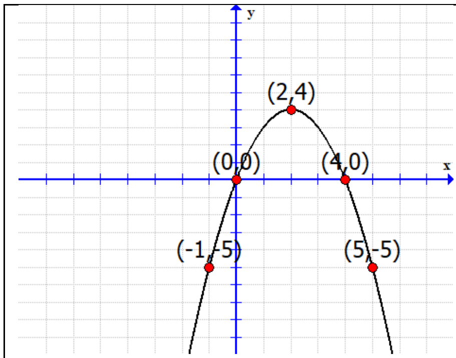
17) Given the graph of g(x), find the following:

a) $g(3)$
b) $g(-3)$
c) all values of x such that $g(x) = 3$
d) all values of x such that $g(x) = 0$



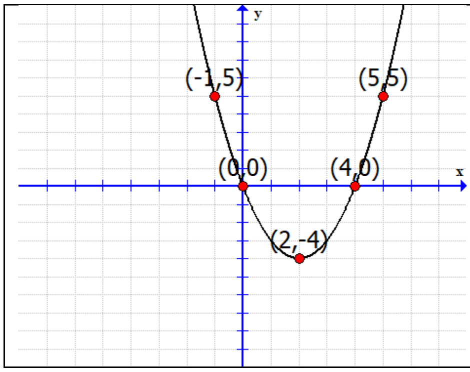
18) Given the graph of g(x), find the following:

a) $g(0)$
b) $g(-4)$
c) all values of x such that $g(x) = -4$
d) all values of x such that $g(x) = 0$



19) Given the graph of h(x), find the following:

a) $h(-1)$
b) $h(2)$
c) all values of x such that $h(x) = -5$
d) all values of x such that $h(x) = 0$



20) Given the graph of h(x), find the following:

a) $h(0)$
b) $h(4)$
c) all values of x such that $h(x) = -4$
d) all values of x such that $h(x) = 0$